Differential equations

(1) Find the solution of diff. eqn.

$$\frac{dy}{dx} = e^{x+y}$$

Soln. Given $\frac{dy}{dx} = e^{x+y}$

$$\Rightarrow \frac{dy}{dx} = e^x \cdot e^y$$

$$\Rightarrow \frac{dy}{e^y} = e^x dx$$

$$\Rightarrow e^x dx = e^{-y} dy$$

Integrating we get

$$\int e^x dx = \int e^{-y} dy$$

$$\Rightarrow e^x = -e^{-y} + K$$

where K is constant of Integration.

Reqd. solution is

$$e^x = -e^{-y} + K$$

(2) Solve $\frac{dy}{dx} = x+y$

Soln. Given $\frac{dy}{dx} = x+y$ ——— (1)

Put $x+y = dz$

Differentiating w.r. to x, we get

$$1+\frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$$

Putting this in eqn (1)

$$\frac{dz}{dx} - 1 = z$$

$$\Rightarrow \frac{dz}{dx} = 1+z$$

$$\therefore \frac{dz}{1+z} = dx$$

Integrating, we get

$$\int \frac{dz}{1+z} = \int dx$$

$$\Rightarrow \log(1+z) = x+K$$

$$\Rightarrow \log(1+x+y) = x+K \quad ; \text{ where } K \text{ is constant of integration.}$$